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Relationship Between Instability Waves and Noise of High-Speed Jets

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The relationship between the instability waves and noise of hot jets at moderate supersonic Mach number is examined. For very-high-speed jets the instability waves propagate downstream with supersonic velocity relative to ambient sound speed. These fast moving instability waves generate intense Mach wave radiation (analogous to the case of supersonic flow past a wavy wall). Under this circumstance the radiated noise characteristics must strongly correlate with those of the instability waves. It is found that the highest sound-pressure level of the far-field noise occurs at a direction and frequency that closely match the Mach wave radiation direction and frequency of the most amplified instability wave of the jet. When the jet speed exceeds the sum of the sound speed inside and outside the jet, the jet flow supports not only the familiar Kelvin-Helmholtz instability waves but also a family of supersonic instability waves. Present calculations show that for jet Mach number up to 2.0 and jet total temperature to ambient temperature ratio up to 2.5, the Kelvin-Helmholtz instability waves always grow to a higher amplitude than the supersonic instability waves. Numerical results indicate that for hot jets the most amplified wave invariably belongs to the helical mode Kelvin-Helmholtz instability wave. For lower speed hot jets with jet static temperature higher than or equal to the ambient temperature there is also a fair correlation between the Strouhal number at the peak sound-pressure level of the far-field noise and that of the most amplified instability wave.

I. Introduction

DURING the last decade much effort has been devoted to the determination of the dominant noise sources of supersonic jets. Presently the consensus is that the noise generated directly by the large turbulence structures/instability waves of the jet flow probably constitute the principal part of the turbulent mixing noise component. There is direct and indirect experimental and theoretical evidence supporting this proposition.[1-4] However, most of the previous works and experiments are confined to cold supersonic jets, whereas jets issued from aircraft engines are invariably hot. For the proposed second generation high-speed civil transport, the operating jet temperature is even higher. The main objective of this work is to examine the relationship between the instability waves/large turbulence structures and noise of hot supersonic jets within the theoretical framework of Ref. 1 and to provide evidence that a reasonable correlation exists between the characteristics of the most amplified instability waves of hot supersonic jets and their radiated noise.

For cold supersonic jets in the Mach number range of 1.0 to 3.0 the large turbulence structures/instability waves of the jets are associated with the well-known Kelvin-Helmholtz instability. Excellent optical observations of these instability waves in supersonic jets can be found in the work of Lepicovsky et al.[5] Recently Oertel[6,7] and Tam and Hu[8] studied the large turbulence structures/instability waves of hot supersonic jets. They found that at jet operating conditions for which the jet velocity was larger than the sum of the speeds of sound inside and outside the jet, the jet could support not only the familiar Kelvin-Helmholtz instability waves but also a new family of instability waves. The new waves are referred to as supersonic instability waves because their phase velocity is greater than the ambient speed of sound. The optical observations of Oertel clearly indicate that both the Kelvin-Helmholtz instability waves and the supersonic instability waves are capable of generating noise. The experimental and theoretical studies by Oertel and Tam and Hu, however, were confined to jets with thin mixing layers. At the present time very little is known about the characteristics of the supersonic instability waves in jets with realistic mixing layer profiles. In this work, both the Kelvin-Helmholtz instability waves and the supersonic instability waves are considered. It will be shown numerically that for jet Mach numbers up to 2 and jet-to-ambient temperature ratios up to 2.5 the Kelvin-Helmholtz instability waves are the dominant instabilities. They are, therefore, expected (as in the case of cold supersonic jets) to be the dominant sources of jet mixing noise.

The processes by which instability waves generate acoustic radiation have been discussed at some length in Ref. 1. It was pointed out that the phase velocity of an instability wave is crucial to how efficiently sound, in the form of Mach waves, is produced. For hot jets at moderate supersonic Mach number, the jet velocity is high so that the instability waves generally propagate downstream at supersonic velocities relative to the ambient sound speed. When moving with supersonic velocity, an instability wave will generate intense Mach wave radiation (analogous to supersonic flow past a wavy wall) with a peak noise direction normal to the Mach wave fronts. For cold or low Mach number supersonic jets, the instability wave speeds are generally subsonic. In this case, the sound generation processes are less direct and less efficient. It has been elaborated in Ref. 1 that even a subsonic wave could cause acoustic radiation due to the growth and decay of the wave amplitude. Because of the growth and decay of the wave, its

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wave number spectrum is broadened so that a subsonically moving wave could still have supersonic wave components. These supersonic wave components would again lead to Mach wave radiation.

In this paper, attention is focused on hot supersonic jets at moderate supersonic Mach numbers. According to the above stated noise generation mechanism, one would expect the characteristics of the dominant part of the noise of these jets to correlate well with those of the most amplified instability waves of the jet flow. For instance, there should be a close match of the frequencies between noise and instability waves. In Sec. III it is shown that reasonably good correlation between the calculated instability wave characteristics and the measured jet noise directivities and spectra of Ref. 9 can, indeed, be found. The correlation, however, deteriorates as expected, for lower speed jets.

II. The Most Amplified Instability Wave

If instability waves are, in fact, the dominant source of hot supersonic jet noise, then the radiated noise characteristics in the direction of maximum noise radiation must be directly related to those of the most amplified instability wave. In this section, a simple procedure for determining the most amplified instability wave under the locally parallel flow approximation is outlined. Some of the characteristics of the most amplified instability wave of hot jets at moderate supersonic Mach number are investigated.

Linear instability wave theory of supersonic jets is now well known. The mathematical and computational steps needed to calculate the spatial growth rates of these waves are fully documented in the literature (see Refs. 1 and 8) and, therefore, will not be repeated here. In performing a linear instability wave calculation for a round jet it is customary and advantageous to use a cylindrical coordinate system (r, θ, x) centered at the nozzle exit (Fig. 1) with the x-axis pointing in the direction of flow. A solution of the governing linearized compressible equations of motion in the form $p(r, \theta, x, t) = \hat{p}(r) \exp[i(kx + n\theta - \omega t)]$ and similar form of solution for the other flow variables is sought. Here ω is the angular frequency of the wave and n is the azimuthal Fourier mode number ($n = 0, 1, 2, \ldots$) and k, the spatial wave number, is the eigenvalue of the problem. That is the eigenvalue solution gives k as a function of ω and n. The imaginary part of k or k_i is the (negative) local growth rate of the wave. Because of the change in velocity and density profiles as the wave propagates downstream, k_i varies with the downstream distance. The total amplification $A(\omega, n)$ of an instability wave of a fixed frequency ω and mode number n for a supersonic jet with Mach number M_j and jet total temperature to ambient temperature ratio T_r/T_a is, therefore, given by:

$$A(\omega, n) = \exp\left[\int_0^{x_c} - k_i(\omega, n, x)\, \mathrm{d}x\right] \tag{1}$$

where x_c is the location at which the wave reaches its maximum amplitude or its local growth rate is zero. In other words $k_i(\omega, n, x_c) = 0$. In Eq. (1) the effect of nonparallel flow has been neglected so that it is only a first estimate. A detailed procedure to account for nonparallel flow correction is given in Ref. 1. To find the most amplified wave it is necessary to identify the mode number and the frequency or Strouhal number of the wave with the highest total growth.

It is known that a supersonic jet can be divided into three regions; the core, the transition, and the fully developed regions according to the characteristics of the mean velocity profile (see Fig. 1). The core region is closest to the nozzle exit. In the core region, the center part of the jet has a uniform velocity equal to the fully expanded jet velocity. Surrounding the core is a mixing layer that broadens in the downstream direction. The mean velocity profile in the core region can be characterized by a two parameter function:

$$\frac{u(r, x)}{u_j} = \begin{cases} 1, & r \le h \\ \exp\left[-\ell n 2\left(\dfrac{r-h}{b}\right)^2\right], & r \ge h \end{cases} \tag{2}$$

where u_j is the fully expanded jet velocity, h is the core radius, and b is the half-width of the shear layer. h and b both vary in the downstream direction. This velocity profile is shown in Fig. 2. The parameters h and b are not independent. They satisfy the requirement of conservation of momentum flux i.e.

$$\int_0^\infty \rho u^2 r\, \mathrm{d}r = \frac{\rho_j u_j^2 R^2}{2} \tag{3}$$

so that the mean velocity u may be regarded as a function of a single parameter b alone. In Eq. (3) R and ρ_j are the jet radius and density at the nozzle exit of the perfectly expanded jet. On invoking boundary-layer approximation to the jet mean flow, one finds that the mean pressure across the jet is nearly constant. This together with the Crocco's relation allows the density distribution across any cross section of the jet to be expressed as a function of the jet velocity. The explicit relation is:

$$\frac{\rho_j}{\rho} = \left(1 + \frac{\gamma - 1}{2} M_j^2\right)\left[\frac{T_a}{T_r} + \left(1 - \frac{T_a}{T_r}\right)\frac{u}{u_j}\right] - \frac{\gamma - 1}{2} M_j^2 \left(\frac{u}{u_j}\right)^2 \tag{4}$$

where M_j is the fully expanded jet Mach number; γ is the specific heat ratio of the gas; and T_r and T_a are the reservoir and ambient temperature respectively. Eqs. (2), (3), and (4) suggest that the velocity and density distribution in the core region of the jet needed for the determination of k_i in Eq. (1) through standard instability wave analysis are completely specified by the single parameter b, the half-width of the jet mixing layer. In other words, the dependence of k_i on x is through its dependence on b; that is, $k_i = k_i(\omega, n, b(x))$.

Extensive jet instability calculations over a significant range of Strouhal number have been carried out. It is found that

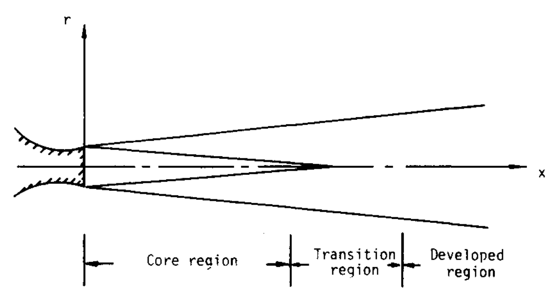

Fig. 1 Schematic diagram of a perfectly expanded supersonic jet divided into the core, the transition and the developed regions.

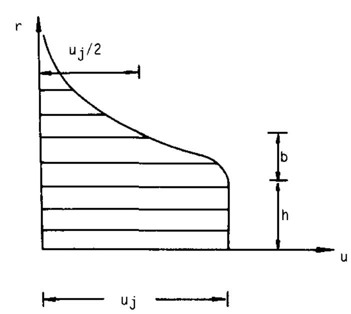

Fig. 2 Velocity profile in the core region of the jet.

the position of neutral instability x_c is invariably located in the core region of the jet. For the purpose of determining the most amplified instability wave it is, therefore, not necessary to consider the transition and the developed regions of the jet at all. A careful examination of all the measured mean jet velocity profiles reveals that b varies nearly linearly with x. This is consistent with the standard similarity argument. In the literature, $db/dx = \sigma$ is referred to as the spreading rate. With σ taken as a constant the total growth integral in Eq. (1) may be cast into the following computationally more convenient form:

$$\int_0^{x_c} - k_i(\omega, n, x)\,dx = \frac{1}{\sigma}\int_{b_0}^{b_c} - k_i(\omega, n, b)\,db \qquad (5)$$

where b_0 is the half-width of the jet mixing layer at the nozzle exit and $b_c = b(x_c)$. Thus, the most amplified instability wave is the one with Strouhal number $S(S = fD/u_j,\ D =$ jet diameter) and mode number n, which maximizes the integral:

$$I = \int_{b_0/R}^{b_c/R} - k_i(S, n, b/R)R\,d(b/R) \qquad (6)$$

Eq. (6) is the same integral as in Eq. (5) except that it is now written in dimensionless form.

Numerical computations of the total growth integral (Eq. (6)) for the Kelvin-Helmholtz instability waves for jets with Mach number M_j up to 2.0 and total temperature to ambient temperature ratio T_r/T_a up to 2.5 have been carried out. The local value of k_i is found by solving the Rayleigh equation as in Refs. 1 and 8. Figure 3 shows a typical set of results for a Mach 1.4 jet at $T_r/T_a = 2.1$. In this figure, the total growths of the $n = 0$, 1, and 2 mode instability waves over a wide range of Strouhal numbers are given. It is evident from this figure that the $n = 1$ mode is preferentially amplified. It turns out that this is always true for hot jets at moderate supersonic Mach number. Thus, for the purpose of identifying the most amplified instability wave only the helical mode $n = 1$ needs to be considered. In Fig. 3 the $n = 1$ mode curve peaks at a Strouhal number of 0.15. Therefore, for this jet the helical instability wave mode at a Strouhal number 0.15 is the most amplified wave.

When the velocity of the jet is sufficiently high such that $u_j > a_j + a_\infty$ where a_j and a_∞ are the speed of sound inside and outside the jet, the jet can support not only the familiar Kelvin-Helmholtz instability waves but also the supersonic instability waves.[6-8] The principal characteristics of the supersonic instability waves have been investigated in Ref. 8. The spatial growth rates of these waves can be calculated in exactly the same way as for the Kelvin-Helmholtz instabilities. One important difference between the Kelvin-Helmholtz instability waves and the supersonic instability waves is that the former is an inflexional instability, whereas the latter is not. The significance is that in the inviscid limit the Kelvin-Helmholtz instability is analytically continued into a corresponding family of damped wave, whereas the supersonic instability is continued into a family of neutral waves. What this means is that without taking viscosity (including eddy viscosity) into account the supersonic instability waves would not become damped as they propagate downstream. To be able to identify the most amplified wave it is, hence, essential to include eddy viscosity effect in the analytical model. In all the calculations for the supersonic instability waves performed in this paper, eddy viscosity terms are included (i.e., the local growth rate is determined by solving the Orr-Sommerfeld equation with eddy viscosity terms). The local eddy Reynolds number $\nu_t b/u_j$ is taken to be equal to 500 (ν_t is the eddy kinematic viscosity). This value of the eddy Reynolds number has been found to be satisfactory for this and similar types of analysis.[10] Figure 4 shows a comparison of the total growth integrals of the most amplified supersonic instability waves (the (0, 1) mode) and those of the Kelvin-Helmholtz instability waves for a Mach 2.0 hot jet at $T_r/T_a = 2.7$. As can be seen, the $n = 1$ and $n = 2$ mode of the Kelvin-Helmholtz instability waves have significantly higher total growth. The former is the most amplified. Numerical investigation over the range of $1.0 \le M_j \le 2.0$ and $T_r/T_a \le 2.5$ confirms that this is generally true. For the purpose of determining the characteristics of the most amplified instability waves within the above operating conditions only the $n = 1$ Kelvin-Helmholtz instability wave mode is considered.

When the jet velocity is sufficiently high, the phase velocity of the most amplified instability wave would be supersonic relative to the ambient sound speed. Under this condition the instability wave would generate intense Mach wave radiation. The principal direction of acoustic radiation is normal to the Mach wave fronts as shown in Fig. 5. If instability waves are

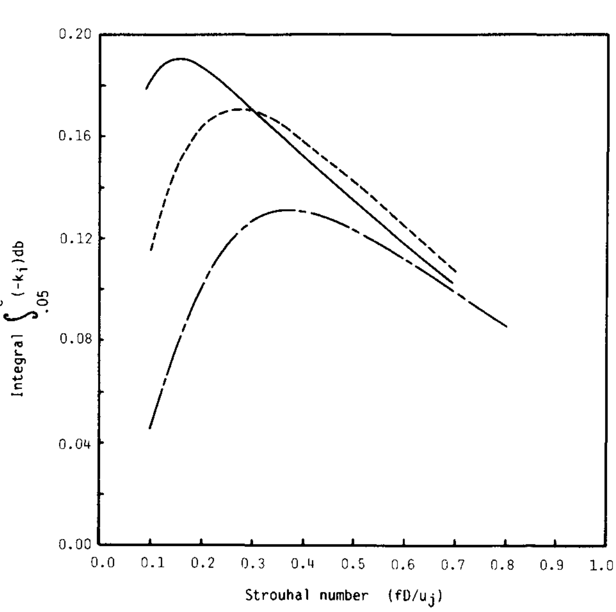

Fig. 3 Total growth integral for the instability waves of a Mach 1.4 jet at $T_r/T_a = 2.1$. — · — · —, mode 0; ——, mode 1; — — — — —, mode 2.

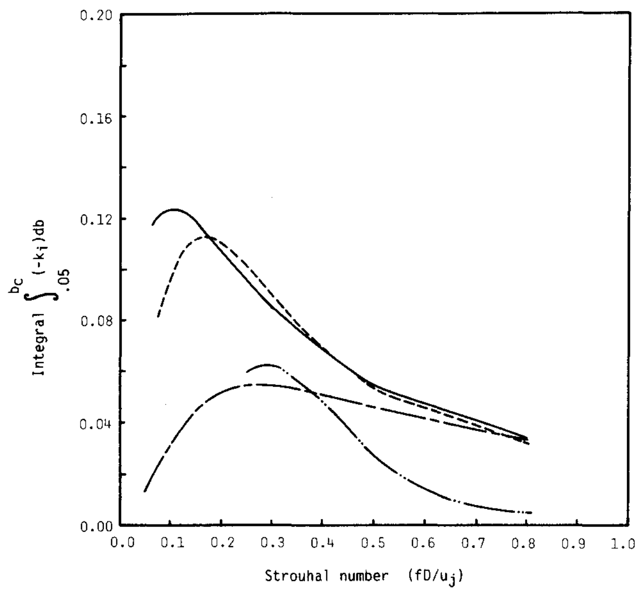

Fig. 4 Total growth integral for the instability waves of a Mach 2.0 jet at $T_r/T_a = 2.7$. — · — · —, mode 0; ——, mode 1; — — — — —, mode 2; — · · — · · —, mode (0,1), supersonic instability waves.

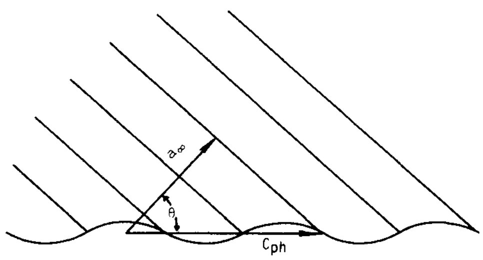

Fig. 5 Vector diagram of Mach wave radiation. $\cos\theta = a_\infty / c_{ph}$.

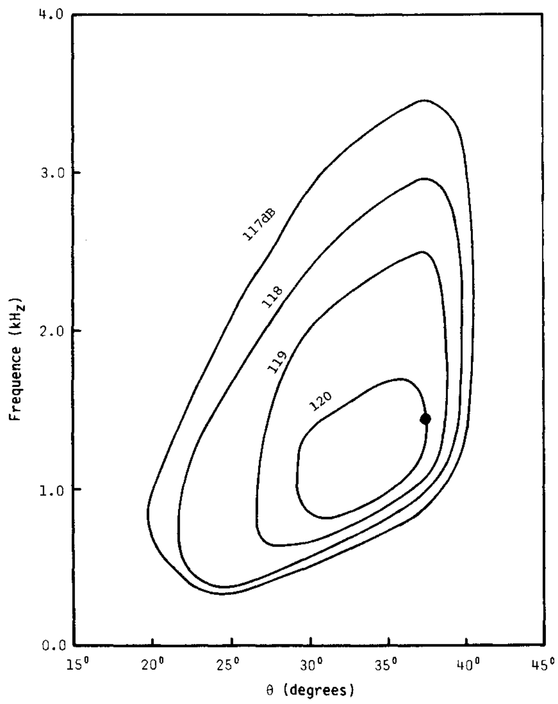

Fig. 6 Comparison between the calculated frequency and direction of Mach wave radiation of the most amplified instability wave of a Mach 2.0 jet at a total temperature of 855°F and measurements. Shown are contours of equal sound-pressure-level in the θ-frequency plane. ●, theoretical value.

the dominant noise source of the jet, then the peak noise direction θ should be given approximately by the formula:

$$\theta = \cos^{-1}(a_\infty / c_{ph}) \tag{7}$$

where c_{ph} is the phase velocity of the most amplified instability wave. By comparing the calculated values of θ and the Strouhal number of the most amplified wave with the far-field noise measurements it is possible to test the validity of the "instability waves are the dominant source of high-speed jet noise hypothesis." This is reported in the next section.

III. Comparison with Experiment.

Good quality, narrow-band, hot supersonic jet noise data are not readily available in the literature. Some years ago Tanna et al.[9] measured the far-field noise of these jets issued from three nozzles of design Mach number 1.4, 1.7, and 2.0. The data were reported in tabulated form in ⅓-octave-band sound-pressure-level. The measurements were carried out by microphones spaced at 7½ degrees apart. For lack of better quality and more complete data set, the Tanna et al. measurements are used for comparison with calculated results. Because ⅓-octave-band data are weighted in favor of higher frequency noise, the sound-pressure-level spectra are first converted into 100-Hz band data before comparisons are made.

Two types of comparisons are carried out. For jets with high enough velocity such that the phase velocity of the most amplified instability wave (at maximum amplitude) is supersonic relative to the ambient sound speed, direct Mach wave radiation is expected. In this case, the Strouhal number and the direction of maximum noise radiation in the far field should be nearly equal to the Strouhal number of the most amplified instability wave and its Mach wave radiation angle. Another set of comparisons are made for jets of lower speed. For these lower speed jets, the phase velocities of the instability waves are subsonic relative to the ambient sound speed. Direct Mach wave radiation, as in the case of supersonic flow past a wavy wall, is not possible. However, acoustic radiation is still being generated due to the growth and decay of the wave amplitude. It is expected that the Strouhal number at the highest sound-pressure-level of the radiated noise would still be nearly equal to that of the most amplified instability wave of the jet. However, such correlation would deteriorate as the jet velocity decreases.

Figure 6 shows the contours of equal sound-pressure-level in dB (100-Hz band) in the frequency-direction of radiation plane of a Mach 2.0 jet at a total temperature of 855°F constructed from the measurements of Tanna et al.[9] Also shown is the calculated frequency and Mach wave radiation angle of the most amplified instability wave. On considering that the peak of the noise spectrum and directivity is quite broad the agreement between calculated result and measurement seems to be quite good. Figures 7 and 8 are further comparisons between theory and measurements for the same Mach 2.0 jet, but at lower jet total temperature. Figure 7 is at $T_r = 470$°F, while Fig. 8 is at room temperature $T_r = 74$°F. The agreement between theory and experiment is again reasonably good. The difference between the measured and calculated frequency is roughly within one ⅓-octave band. The difference between the measured and calculated peak noise direction is around 5 deg, which is less than the spacing of the microphones used in acquiring the data.

In Fig. 9, the measured Strouhal numbers ($S = fD/u_j$ where $D = 2R$ is the diameter of the jet at the nozzle exit) corre-

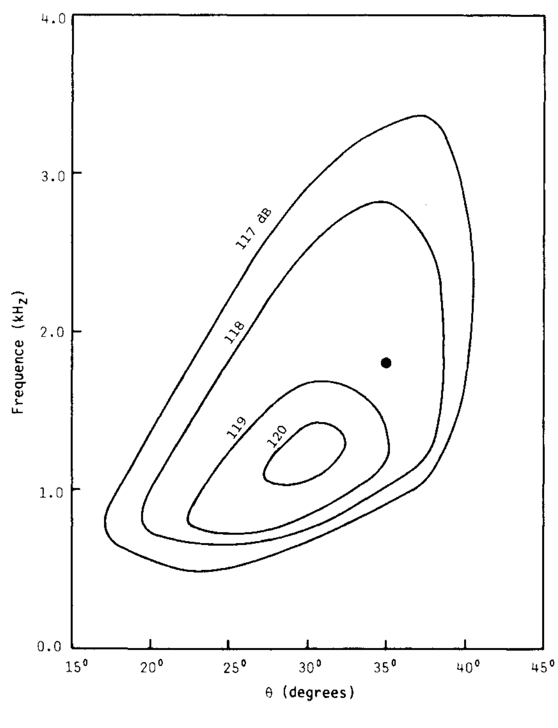

Fig. 7 Comparison between the calculated frequency and direction of Mach wave radiation of the most amplified instability wave of a Mach 2.0 jet at a total temperature of 470°F and measurements. Shown are contours of equal sound-pressure-level in the θ-frequency plane. ●, theoretical value.

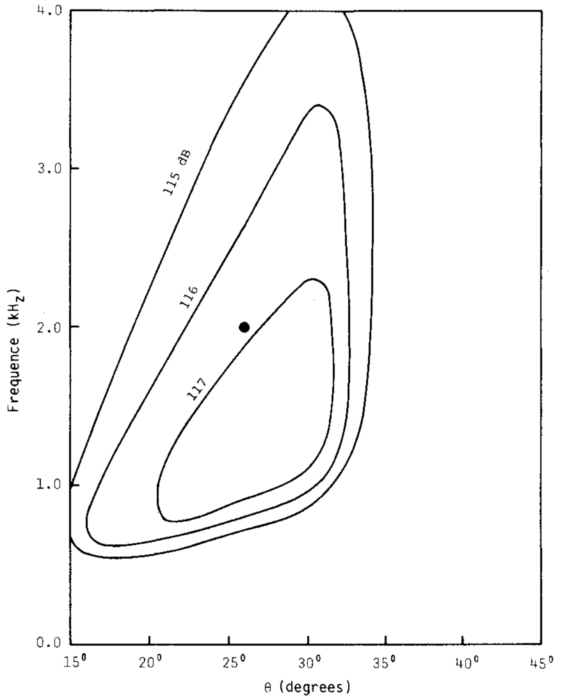

Fig. 8 Comparison between the calculated frequency and direction of Mach wave radiation of the most amplified instability wave of a Mach 2.0 jet at a total temperature of 74°F and measurements. Shown are contours of equal sound-pressure-level in the θ-frequency plane. ●, theoretical value.

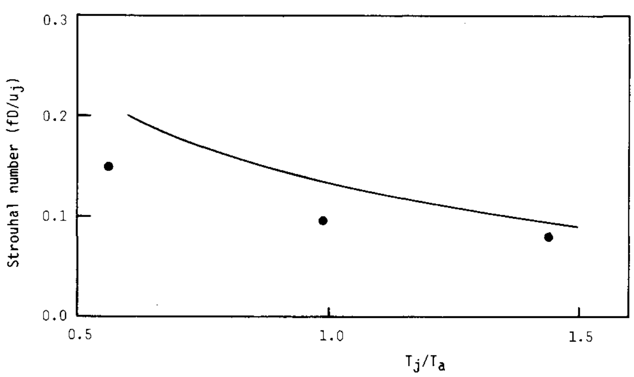

Fig. 9 The Strouhal number at maximum sound-pressure-level of a Mach 2.0 jet as a function of jet to ambient temperature ratio. —, theory; ●, experiment.

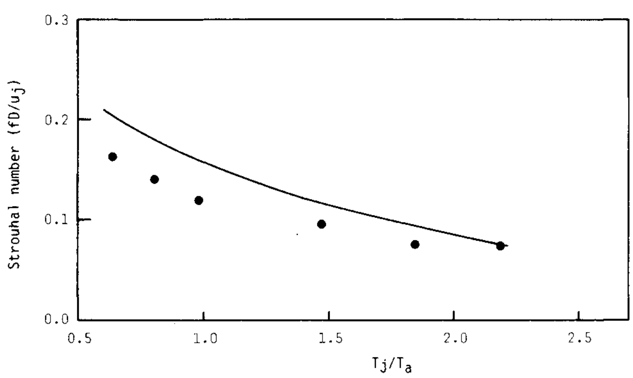

Fig. 10 The Strouhal number at maximum sound-pressure-level of a Mach 1.7 jet as a function of jet to ambient temperature ratio. —, theory; ●, experiment.

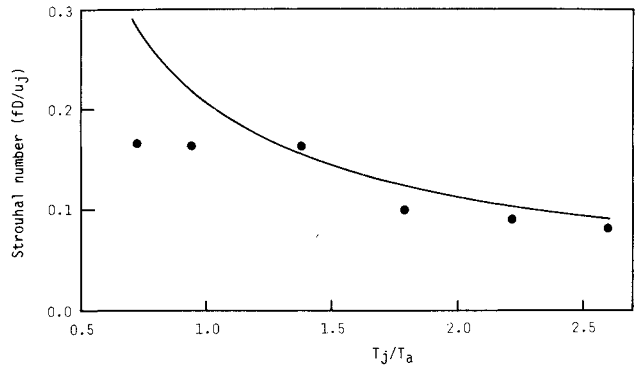

Fig. 11 The Strouhal number at maximum sound-pressure-level of a Mach 1.4 jet as a function of jet to ambient temperature ratio. —, theory; ●, experiment.

sponding to the highest far-field sound-pressure-level of a Mach 2.0 jet operating at different jet to ambient temperature ratios are compared with the Strouhal numbers of the most amplified instability waves. Similar comparisons for the Mach 1.7 and Mach 1.4 jets are given in Figs. 10 and 11. For hot jets with jet temperature higher than that of the ambient, there seems to be generally fair to good agreement. The agreement seems to be better at higher jet temperatures and jet velocity. For cold jets ($T_j < T_a$), the agreement is less good, especially at lower jet Mach number. This trend is consistent with the dependence of the Mach wave noise radiation mechanism on the jet speed discussed above and in Ref. 1.

IV. Summary

In this paper the source of hot jet noise at moderate supersonic Mach number is investigated. The relative importance of the Kelvin-Helmholtz instability waves and the supersonic instability waves as sources of noise is assessed. It is found that for the range of Mach number and jet to ambient temperature ratio considered the Kelvin-Helmholtz instability waves have much higher total amplification and higher phase speed, thus, (just as for cold jets) they are the more dominant source for hot jet noise. According to the instability wave noise generation mechanism discussed in Ref. 1, it is expected that for high-speed jets, especially those with supersonically traveling (relative to the ambient speed of sound) instability waves, the Strouhal number and the direction of maximum noise radiation should correlate well with the Strouhal number and the direction of Mach wave radiation of the most amplified instability wave. Comparisons between calculated results and the experimental measurements of Ref. 9 are made. Reasonably good agreements are found. For lower speed hot jets with $T_j > T_a$, fairly good correlations between the measured Strouhal number at maximum far-field sound-pressure-level and the calculated Strouhal number of the most amplified instability wave are also observed. The correlations deteriorate progressively as the jet speed is reduced. It is believed that this is because at subsonic phase speed, the intensity of Mach wave radiation from an instability wave does not simply depend upon its amplitude alone. The entire history of the spatial evolution of the instability wave (growth and decay) needs to be taken into account in determining the radiated noise intensity. For very low speed jets, the instability waves are probably not the dominant direct source of noise.

Acknowledgment

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References

[1]Tam, C. K. W., and Burton, D. E., "Sound Generated by Instability Waves of Supersonic Jets. Part 1, Two Dimensional Mixing Layers. Part 2, Axiymmetric Jets," *Journal of Fluid Mechanics*, Vol. 138, 1984, pp. 249–271, 273–295.

[2]Troutt, T. R., and McLaughlin, D. K., "Experiments on the Flow and Acoustic Properties of a Moderate Reynolds Number Supersonic Jet," *Journal of Fluid Mechanics*, Vol. 116, 1982, pp. 123–156.

[3]Seiner, J. M., McLaughlin, D. K., and Liu, C. H., "Supersonic Jet Noise Generated by Large-Scale Instabilities," NASA TP-2072, 1982.

[4]Tam, C. K. W., "Jet Noise Generated by Large-Scale Coherent Motion," *Aeroacoustics of Flight Vehicles, Theory and Practice*, edited by H. H. Hubbard, Vol. 1, NASA RP-1258, pp. 311–384, 1991.

[5]Lepicovsky, J., Ahuja, K. K., Brown, W. H., and Burrin, R. H., "Coherent Large-Scale Structures in High Reynolds Number Supersonic Jets," NASA CR-3952, 1985.

[6]Oertel, H., "Coherent Structures Producing Mach Waves Inside and Outside of the Supersonic Jet," *Structure of Complex Turbulent Shear Flow*, International Union of Theoretical and Applied Mechanics Symposium, Marseille, 1982.

[7]Oertel, H., "Mach Wave Radiation of Hot Supersonic Jets Investigated by Means of the Shock Tube and New Optical Techniques," *Proceedings of the Twelfth International Symposium on Shock Tubes and Waves*, Jerusalem, 1980, pp. 266–275.

[8]Tam, C. K. W., and Hu, F. Q., "On the Three Families of Instability Waves of High Speed Jets," *Journal of Fluid Mechanics*, Vol. 201, 1989, pp. 447–483.

[9]Tanna, H. K., Dean, P. D., and Burrin, R. H., "The Generation and Radiation of Supersonic Jet Noise. Volume III, Turbulent Mixing Noise Data," United States Air Force Aero Propulsion Laboratory Report AFAPL-TR-76-65-Vol III, 1976.

[10]Tam, C. K. W., Jackson, J. A., and Seiner, J. M., "A Multiple-Scales Model of the Shock-Cell Structure of Imperfectly Expanded Supersonic Jets," *Journal of Fluid Mechanics*, Vol. 153, 1985, pp. 123–149.